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AUTHOR Campbell, Kathleen T.
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ABSTRACT

Advantages of the use of multivariate commonality analysis are discussed and a small data set is used to illustrate the analysis and as a model to enable readers to conduct such an analysis. A noteworthy advantage of commonality analysis is that commonality honors the relationships among variables by determining the degree to which predictors in a set share variance with the criterion variables. Since commonality indicates the extent of overlap of the variables, it is especially useful in the behavioral sciences where predictor variables are often correlated with each other. Commonality also reinforces the recognition that canonical analysis is the most general case of parametric significance testing. The disadvantage that there are no statistical significance tests for commonality analyses is outweighed by the advantages. The illustrative example uses a hypothetical data set of 22 observations, 3 predictor variables, and 2 criterion variables. Seven data tables are provided. (SLD)

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METHODS OF MULTIVARIATE COMMONALITY ANALYSIS

Kathleen T. Campbell

University of New Orleans 70148

Paper presented at the annual meeting of the Southwest Educational Research Association, Houston, TX, January 25, 1990.

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ABSTRACT

Commonality or variance partition analysis has been available to researchers as a very valuable supplement to regression analysis for some time (Newton & Spurrell, 1967). These methods were popularized in the well known Coleman Equal Opportunity studies. Seibold and McPhee (1979) presents an example analysis in a cancer study in which regression results might have been seriously distorted if the researchers had not conducted a commonality analysis.

However, the same analytic logic can be applied in techniques other than regression. For example, canonical correlation analysis subsumes regression as a special case, so logically the method may be useful in multivariate analyses as well (Thompson & Miller, 1985).

The paper illustrates how a multivariate commonality analysis is conducted. A small data set is used to make the discussion concrete. Thus, the paper will enable readers to conduct their own multivariate commonality analysis using the examples as a model.

METHODS OF MULTIVARIATE COMMONALITY ANALYSIS

In a 1988 paper presented at the annual meeting of the American Educational Research Association, Thompson demonstrated that canonical correlation analysis, and not multiple regression, is the most general linear model, subsuming both univariate and multivariate parametric methods as special cases. Following Knapp (1978), and using a hypothetical data set, Thompson (1988) illustrated how canonical analyses give the same results as t tests, and correlation, ANOVA, MANOVA, multiple regression, and discriminant analyses. He suggested, since canonical correlation analysis subsumes multiple regression as a special case, and since commonality analysis has proven helpful in interpreting multiple regression results (Thompson & Borrello, 1985), that interpretation of canonical results would likewise be facilitated with the use of commonality analysis. According to Daniel (1989, pp. 5-6), "the unique explanatory power of individual independent variables in a multivariate data set" is a significant issue which is addressed by multivariate commonality analysis. Likewise, Beaton (1973, p. 38) stressed the importance of commonality analysis, stating that it "is a technique for assessing the common and unique predictability of several regressors or sets of regressors on a set of p $\geq$ 1 regressands."

In the present paper, a canonical correlation analysis using a hypothetical data set of 22 observations, consisting of three predictor variables and two criterion variables, is interpreted. A commonality analysis of the data is then explained and illustrated. These results are interpreted in conjunction with the canonical analysis run on the data set.

The hypothetical data set involved 22 cases or observations. The variables were opinions (on a scale of 1-20) about various events which occurred during the Reagan administration. Three variables were designated as predictor variables: LESSFED (less federal aid), LESSWEL (less welfare spending), and MOREDEF (more defense spending). Two variables were designated criterion variables: MORESS (more spending on social security) and CATMED (catastrophic medicine insurance coverage). Table 1 presents the hypothetical data. Table 2 presents descriptive statistics for the data.

Insert Tables 1 and 2 about here.

According to Thompson (1988, p. 3), "[C]anonical analysis, like all parametric methods, involves the creation of 'synthetic' scores for each person." The synthetic scores in canonical analysis are the composite scores on each canonical function: a weighted criterion composite and a weighted predictor composite. Although the SAS program calculates the standardized z-scores and the synthetic

composites automatically through the PROC CANCORR command, both to illustrate what canonical correlation is and to perform a subsequent commonality analysis, the synthetic composite scores (also called variate scores) were computed here within the SAS program presented in Appendix A. In steps 6-10 of the program z-scores were computed. These z scores were required to create the synthetic variables, criterion composite scores (C1 and C2) and predictor composite scores (P1 and P2), in the data set. Steps 11-14 in the program show the algorithm for creating these variables: each z-score is weighted by a function coefficient analogous to a beta weight, and these weighted products are then summed. The function coefficients were derived from the canonical correlation results presented in Table 3. The various correlations (steps 25 to 48) requested in Appendix A command file were computed to illustrate what structure coefficients, index coefficients, and canonical correlations really are. The last commands (steps 49 to 51) in Appendix A are multiple regressions on the synthetic criterion variable scores (C1 and C2) using every possible combination of the predictors in the predictor variable set. This SAS procedure, PROC RSQUARE, was very convenient in generating these results, an essential component of the multivariate commonality analysis.

Insert Table 3 about here.

As indicated in Table 3, the canonical correlation of the predictors and the criterion variables for the first function was .852653, and the squared canonical correlation was .727017. In other words, the synthetic predictor variable scores (P1) on the first function accounted for almost 73% of the variance of the synthetic criterion variable scores (C1). On the second function, however, the squared canonical correlation was only .283616, accounting for only 28% of the variance. The first likelihood ratio, lambda, was .25, and the F was statistically significant at the .0004 alpha level. The second function was not significant ($p > .4701$).

As Thompson notes (1988, p. 6), the "canonical correlation ($\underline{R}c$) is nothing more (or less) than the Pearson product-moment correlation between the synthetic variable scores of the subjects on a given function." To reinforce the meaning of the canonical correlation, a correlation between the predictor composite scores (P1) and the criterion composite scores (C1) was also run (Appendix A program lines 25-26), and of course the result is equivalent to the $\underline{R}c$ value, as expected.

The first function for the very small data set of 22 was statistically significant at a very stringent alpha level (.0004). So, while statistical significance is a function of sample size (i.e., as n increases, so does the probability of finding statistical significance), the fact that results for a very small data set were significant is an argument for its

importance. Tests of statistical significance, however, are being used more and more by researchers merely as a minimal criterion "to use in deciding which canonical functions to interpret" (Thompson, 1984, p. 20). Since only the first function was significant, the interpretations that follow will concern the first function only.

The standardized canonical function coefficients reported in Table 3 are the synthetic weights used to multiply the standardized z-scores to compute the composite variate scores. Thus, on the first function, the composite criterion variable set was calculated using the formula C1 = (1.5059*ZMSS) + (-.6529*ZCTMD), as shown in step 11 in Appendix A. The formula for calculating the composite predictor variate score was P1 = (2.2983*ZLSFD) + (.9122*ZLSWL) + (2.3081*ZMDF) as shown in step 13 in Appendix A.

Canonical structure coefficients are reported in Table 3. According to Thompson (1984) and Thompson and Borrello (1985), structure coefficients are particularly important; they serve as an invaluable aid in "interpreting canonical results in terms of each variable's contribution to the canonical solution" (Thompson, 1984, p. 24). The structure coefficients of the criterion variables are the correlation coefficients between the criterion variables and the synthetic criterion composite scores (C1 or C2); similarly, the structure coefficients of the predictors are the correlation coefficients between the predictors and their

synthetic predictor composite scores (P1 or P2). To reinforce the meaning of the structure coefficients, a correlation between the z-score of each variable and the synthetic composite scores of the set to which each variable belonged was also run. These commands are listed in steps 29 38 in Appendix A. The results are the same as those reported in Table 3, computed also in the PROC CANCORR procedure (steps 19-21). For example, the structure coefficient of MORESS on the first function is .9434; the correlation between ZMSS and C1 is .94375.

Index coefficients are also reported in Table 3. The index coefficient is a measure of the relationship between a variable and the synthetic composite scores of the variable set to which the variable does not belong. To reinforce the meaning of the index coefficients, a correlation between the z-score of each variable and the composite set to which each did not belong was also computed. The commands for these procedures are listed in steps 39-48 in Appendix A, and identical results from PROC CANCORR were presented in Table 3. For example, the index coefficient of MORESS with Pred 1 as computed by CANCORR is .8044, while the correlation between ZMSS and P1 as computed by PROC CORR is .80440.

One can use the structure coefficients to calculate the adequacy and the redundancy coefficients in order to enhance interpretation of the canonical analysis, and these are also reported in Table 3. The adequacy coefficients are determined by squaring the structure coefficients for each

variable, then adding across the rows, and then dividing by the number of variables to get the average. This average, or adequacy coefficient, is a measure of "how much of the variance in the variables, on the average, is contained within the synthetic scores for that function" (Thompson, 1988, pp. 18-19). Stewart and Love (1968) suggested multiplying the adequacy coefficient by the squared Rc to yield a redundancy coefficient (Rd).

But according to Thompson (1988), interpretation of the redundancy coefficient is not very useful in a conventional canonical analysis, for several reasons. First, as Cramer and Nicewander (1979) have proved, redundancy coefficients are not truly multivariate, a fact which makes interpretation of redundancy coefficients disturbing. One uses multivariate methods in order to consider all relationships simultaneously (Thompson, 1988), not to find the equivalent of a set of univariate results which consider only one dependent variable at a time. Second, it is contradictory to use an analysis which uses function coefficients to optimize the Rc, but then to interpret results which are not optimized as part of the analysis, such as redundancy coefficients (Thompson, 1988).

However, commonality analysis, according to Thompson (1988), does enhance interpretation of canonical results. Thompson and Miller (1985, p. 2) maintain that the "analysis indicates how much of the explanatory power of a variable is 'unique' to the variable, and how much of the variable's explanatory ability is 'common' to or also available from one

or more other variables." The technique may be done in a univariate regression case or in a multivariate canonical correlation analysis. The only difference between the univariate and the multivariate analysis is that in the multivariate case the criterion variables must be converted to a composite score, i.e., the synthetic composite score (Daniel, 1989). The steps used to conduct multivariate commonality analysis are, briefly, as follows: (a) run a canonical correlation analysis on the data; (b) calculate the z-scores and the criterion composite scores; (c) calculate the regression equations, using all possible combinations of predictors to predict the synthetic criterion composite scores; (d) calculate the unique and common variance effects, and then add the columns for each predictor variable to find the sum of the explanatory power of each predictor. In the present case, a commonality analysis was conducted only on the first canonical function. Nevertheless, a commonality analysis could also have been conducted on the second canonical function if one were interested in determining the explanatory power of the predictor variables on that function.

Since a canonical analysis and the z-scores and the composite scores were already computed for illustrative purposes, the first two steps have already been completed. To calculate the Rcs for every possible combination of predictors on the criterion composite, PROC RSQUARE can be run on the data, using C1 as the dependent variable (see

steps 49-50 in Appendix A). PROC RSQUARE is a SAS procedure that will compute regressions on the dependent variable, using every possible combination of predictors--in the present case, singly, doubly, and all three at once. These results are presented in Table 4. According to Daniel (1989), the R-Square, when all three predictors are used in combination, equals the squared canonical correlation. The squared canonical correlation in Table 3 is .727017, within rounding error of the R-Square (.72701639) for all three predictors presented in Table 4.

Insert Table 4 about here.

There are seven possible combinations of the three predictors, labelled predictor sets #1-7 in Table 4. The R_{c}^{2} for each predictor set (derived from the PROC RSQUARE output) is also listed. Table 5 lists the formulas for partitioning the predictor sets and the results. For example, in order to calculate the explanatory ability which is unique to ZLSFD, one must subtract the variance explained by ZLSWL and ZMDF in combination from the variance explained by ZLSFD, ZLSWL and ZMDF in combination. The negative coefficients reported for what is common to ZLSFD and ZLSWL and for what is common to ZLSFD and ZMDF do not indicate that these variables have a less than zero explanatory ability. Negative calculations indicate the presence of suppressor effects, such as negative correlations between variables. Beaton (1973) and Thompson

and Miller (1985) discuss such effects.

Insert Table 5 about here.

After computing these results, the appropriate partitions must be entered under a column for each predictor, as shown in Table 6. Adding the columns for each predictor gives the sum of the partitions for each predictor. This analysis shows that most of the predictive power of the predictor variables is common to all three variables (.55109, or 55%). In other words, although ZMDF alone can predict 63.8789% of the variance in C1, 55.109% of the total explanatory power is common to all three predictor variables. The unique predictive ability of ZMDF is only 9.0224% (63.8789 - 55.109).

Summary

According to Daniel (1989) and Thompson and Miller (1985), there are several noteworthy advantages in conducting commonality analyses. First, commonality honors the relationships among variables by determining the degree to which predictors in a set share variance with the criterion variable(s). Second, since commonality indicates the extent of overlap of the variables, it is especially useful in the behavioral sciences where predictor variables are often correlated with each other. Finally, commonality reinforces the recognition that canonical analysis is the most general case of parametric significance testing. The disadvantage is

that there are no statistical significance tests for commonality analyses; however, since the emphasis in commonality is on interpretation after a statistically significant canonical correlation has been found, this disadvantage is academic, and the advantages outweigh the disadvantage.

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Table 1

Hypothetical Data

	Criterion Variables		Predictors		
OBS	MORESS	CATMED	LESSFED	LESSWEL	MOREDEF
1	16	15	20	19	20
2	14	14	19	20	19
3	13	12	10	10	11
4	13	14	9	10	10
5	15	15	8	10	9
6	14	15	7	8	8
7	16	17	20	20	19
8	15	13	19	20	19
9	16	15	18	20	19
10	16	14	17	18	17
11	12	10	15	15	15
12	11	10	8	7	8
13	9	10	8	8	6
14	15	14	18	18	17
15	13	13	10	10	10
16	15	15	17	18	17
17	16	16	20	19	19
18	15	14	19	19	20
19	14	14	16	15	16
20	13	13	10	10	9
21	12	11	15	14	15
22	12	13	9	10	9

Table 2

Descriptive Statistics

Variable	Mean	Std Dev
MORESS	13.86363636	1.88466126
CATMED	13.50000000	1.92106121
LESSFED	14.18181818	4.84656347
LESSWEL	14.45454545	4.78815277
MOREDEF	14.18181818	4.78724857

Pearson Correlation Coefficients

Prob > /R/ under H0:RH0 = 0 / N = 22

	MORESS	CATMED	LESSFED	LESSWEL	MOREDEF
MORESS	1.00000 0.0000	0.86148 0.0001	0.69100 0.0004	0.74596 0.0001	0.74179 0.0001
CATMED	0.86148 0.0001	1.00000 0.0000	0.45519 0.0333	0.51769 0.0136	0.48672 0.0216
LESSFED	0.69100 0.0008	0.45519 0.0333	1.00000 0.0000	0.98123 0.0001	0.98571 0.0001
LESSWEL	0.74596 0.0001	0.51769 0.0136	0.98123 0.0001	1.00000 0.0000	0.98093 0.0001
MOREDEF	0.74179 0.0001	0.48672 0.0216	0.98571 0.0001	0.98093 0.0001	1.00000 0.0000

Table 3

Canonical Correlation Analysis

	Canonical Correlation	Squared Canonical Correlation	Likelihood Ratio	F	PR > F
1	0.852653	0.727017	0.25102482	5.6435	0.0004
2	0.283616	0.080438	0.91956209	0.7873	0.4701

Standardized Canonical Function Coefficients

	Criterion Variables			Predictor Variables	
	Crit 1	Crit 2		Pred 1	Pred 2
MORESS	1.5059	-1.2691	LESSFED	-2.2983	-2.1816
CATMED	-0.6529	1.8580	LESSWEL	0.9122	5.6358
			MOREDEF	2.3081	-3.5085

Structure Coefficients

	Criterion Variables			Predictor Variables	
	Crit 1	Crit 2		Pred 1	Pred 2
MORESS	0.9434	0.3315	LESSFED	0.8719	-0.1100
CATMED	0.6444	0.7647	LESSWEL	0.9211	0.0535
			MOREDEF	0.9374	-0.1307

Index Coefficients

	Criterion Variables			Predictor Variables	
	Pred 1	Pred 2		Crit 1	Crit 2
MORESS	0.8044	0.0940	LESSFED	0.7434	-0.0312
CATMED	0.5495	0.2169	LESSWEL	0.7853	0.0152
			MOREDEF	0.7993	-0.0371

Table 4

Prediction of Composite (C1) Scores

Using Alternate Predictor Variable Combinations

and PROC RSQUARE

Predictor Set	Variable(s) in Set	$\underline{R}c_2$
1	ZLSFD	.552579
2	ZLSWL	.616796
3	ZMDF	.638789
4	ZLSFD, ZLSWL	.636792
5	ZLSFD, ZMDF	.708459
6	ZLSWL, ZMDF	.638839
7	ZLSFD, ZLSWL, ZMDF	.727016

Note. The $\underline{R}c^2$ for set #7 (using all three predictor variables) equals the squared canonical correlation, within rounding error.

Table 5

Calculations of Unique Variance Partitions

Set	Partition	Result
1	Unique to ZLSFD $-Rc^2$ 6 + Rc^2 7 -.638839 + .727016	.088173
2	Unique to ZLSWL $-Rc^2$ 5 + Rc^2 7 -.708459 + .727016	.018557
3	Unique to ZMDF $-Rc^2$ 4 + Rc^2 7 -.636792 + .727016	.090224
4	Common to ZLSFD and ZLSWL $-Rc^2$ 3 + Rc^2 5 + Rc^2 6 - Rc^2 7 -.638789 + .708459 + .638839 - .727016	-.018507
5	Common to ZLSFD and ZMDF $-Rc^2$ 2 + Rc^2 4 + Rc^2 6 - Rc^2 7 -.616796 + .636792 + .638839 - .727016	-.068181
6	Common to ZLSWL and ZMDF $-Rc^2$ 1 + Rc^2 4 + Rc^2 5 - Rc^2 7 -.552579 + .636792 + .708459 -.727016	.065656
7	Common to ZLSFD, ZLSWL, and ZMDF Rc^2 1 + Rc^2 2 + Rc^2 3 - Rc^2 4 -- Rc^2 5 - Rc^2 6 + Rc^2 7 .552579 + .616796 + .638789 - .636792 -.708459 - .638839 + .727016	.55109

Table 6

Multivariate Commonality Analysis

#	Partition	ZLSFD	ZLSWL	ZMDF
1	Unique to ZLSFD	.088173		
2	Unique to ZLSWL		.018557	
3	Unique to ZMDF			.090224
4	Common to ZLSFD, ZLSWL	-.018507	-.018507	
5	Common to ZLSFD, ZMDF	-.068181		-.068181
6	Common to ZLSWL, ZMDF		.065656	.065656
7	Common to ZLSFD, ZLSWL, and ZMDF	.55109	.55109	.55109
	Sum of Partitions	.55275	.616796	.638789
	r^2 of predictor with canonical composite (C1)	55.28%	61.68%	63.88%

Table 7

Canonical Correlation Analysis Coefficients

Variable/ Coefficient	I Func	Stru	Sq Stru	II Func	Stru	Sq Stru	h^2
LESSFED	-2.298	.872	.76	-2.182	-.110	.0121	.77
LESSWEL	.912	.921	.848	5.636	.054	.003	.85
MOREDEF	2.308	.937	.878	-3.509	-.131	.017	.89
Adequacy	.839						
Redundancy	.603			.001			
$\underline{R}c^2$	.727017			.080438			
Redundancy	.474			.028			
Adequacy	.653			.347			
MORESS	1.506	.943	.889	-1.269	.332	.110	.99
CATMED	-.653	.644	.415	1.858	.765	.585	1.00

APPENDIX A

SAS Commands

```
DATA ONE;
INFILE TTECH;
INPUT PARTY $ 1-3 LESSFED 5-6 LESSWEL  8-9 MOREDEF 11-12
  MORESS 14-15 CATMED 17-18 LESSED 20-21  SDI 23-24
  IRANCON 26-27 HEARING 29-30 INVAR 32-33;
ZMSS=(MORESS-13.86)/1.88;
ZCTMD=(CATMED-13.5)/1.92;
ZLSFD=(LESSFED-14.18)/4.85;
ZLSWK=(LESSWEL-14.45)/4.79;
ZMDF=(MOREDEF-14.18)/4.787;
C1=(1.5059*ZMSS) + (-.6529*ZCTMD);
C2=(-1.269*ZMSS) + (1.858*ZCTMD);
P1=(-2.298*ZLSFD) + (.912*ZLSWL) + (2.308*ZMDF);
P2=(-2.1816*ZLSFD) + (5.6358*ZLSWL) + (-3.5085*ZMDF);
PROC PRINT;
  VAR MORESS CATMED LESSFED LESSWEL MOREDEF;
PROC CORR;
  VAR MORESS CATMED LESSFED LESSWEL MOREDEF;
PROC CANCORR VPREFIX=CRIT WPREFIX=PRED;
  VAR MORESS CATMED;
  WITH LESSFED LESSWEL MOREDEF;
PROC PRINT;
  VAR MORESS CATMED LESSFED LESSWEL MOREDEF ZMSS ZCTMD
  ZLSFD ZLSWL ZMDF C1 C2 P1 P2;
PROC CORR;
 VAR P1 C1;
PROC CORR;
  VAR P2 C2;
PROC CORR;
  VAR ZMSS C1;
PROC CORR;
  VAR ZCTMD C1;
PROC CORR;
  VAR ZLSFD P1;
PROC CORR;
  VAR ZLSWL P1;
PROC CORR;
  VAR ZMDF P1;
PROC CORR;
  VAR ZMSS P1;
PROC CORR;
  VAR ZCTMD P1;
PROC CORR;
  VAR ZLSFD C1;
PROC CORR;
  VAR ZLSWL C1;
PROC CORR;
  VAR ZMDF C1;
PROC RSQUARE;
  MODEL C1=ZLSFD ZLSWL ZMDF;
  MODEL C2=ZLSFD ZLSWL ZMDF;
```